Debt Accumulation and Default in Low-Income Countries

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Abstract

This paper explores the accumulation path of the external public debt of low-income countries (LICs) following debt relief programs. Using the relaxation of IMF debt limits for LICs that received debt relief in the early 2000s, I document that even though LICs initially lowered external debt during debt relief programs, many experienced a fast resurgence in their indebtedness and increased default risk once borrowing limits were lifted. Using a difference-in-differences model, I show that countries that benefited from the relaxation seem more likely to experience a significant increase in their debt-to-GDP ratio. I then evaluate these debt limits policies using a quantitative model of sovereign default that allows for self-fulfilling debt crises. The model includes two types of debt - subsidized loans from multilateral institutions and non-subsidized loans from the private market and an impatient government. After debt limits are lifted, my model predicts that the lower the government discount's factor is compared to the international lenders, the more likely the country is to enter the crisis zone and be in debt distress. I find that having an impatient government from the perspective of a more patient household leads to a decrease in welfare by 0.9%.

Keywords: Debt crisis, low-income countries, sovereign default

JEL Classification: F34, F35, F41, O11

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1 Introduction

In an interview with the Financial Times in 2014, Christine Lagarde at the time, chair and managing director of the International Monetary Fund (IMF) warned African nations against issuing sovereign bonds. She said that instead "governments should be attentive and they should be cautious about not overloading the countries with too much debt. That is additional financing, but that is an additional vulnerability."

Most low-income countries (LICs) have, for a long time, relied and still rely on foreign aid or borrow from multilateral development institutions such as the World Bank and the IMF to finance their needs. In the late 80s and early 90s, many of these LICs faced unsustainably high debt levels. This prompted the creation of debt relief programs (see appendix A) such as the Highly Indebted Poor Countries Initiative (HIPC) and the Multilateral Debt Relief Initiative (MDRI) by the IMF and/or the World Bank. Their goal was to provide debt relief and low-interest loans to cancel or reduce external debt repayments to sustainable levels.

To be part of these programs, the recipient country had to face a borrowing limit on its external non-concessional debt. In response to the critique that having tight debt limits policy puts too much constraint on the LICs, the debt limits policy was relaxed, jointly by the IMF and the World Bank (IMF, 2009). This intervention, combined with the debt relief programs, led to an increase in the borrowing space for these countries. Over the past decade, there has been a change in the composition of the external public debt of the LICs, with an increase in Eurobond issuance by many sub-Saharan African countries (SSA).

Looking at the data, I find that these debt relief initiatives seemed to have partially achieved their goals of reducing debt burdens, but only in the short term. Ten to fifteen years later, many recipient countries of these programs experience a resurgence in their levels of external debt. For instance, Senegal's external public debt-to-GDP ratio went from 54% in 2000 to 14% in 2006, and in 2018 it reached 51%. In 2016, Mozambique defaulted on its infamous "Tuna bonds", bonds issued by Ematum, a government-backed agency in charge of promoting "the fishery activity of tuna" (Figure 1). Mozambique thus became the first African country to default on dollar bonds since Ivory Coast in 2011.[1]

In this light, my project aims to analyze how low-income countries accumulate debt to unsustainable levels and end up defaulting on their debt. It will try to answer the following questions:

[1] *African Issuers Scrutinized After Mozambique's Bond Default* - February 2017 (Bloomberg website)

What are the drivers of debt accumulation in LICs? How impatient are these governments? What are the impacts of said impatience on social welfare?

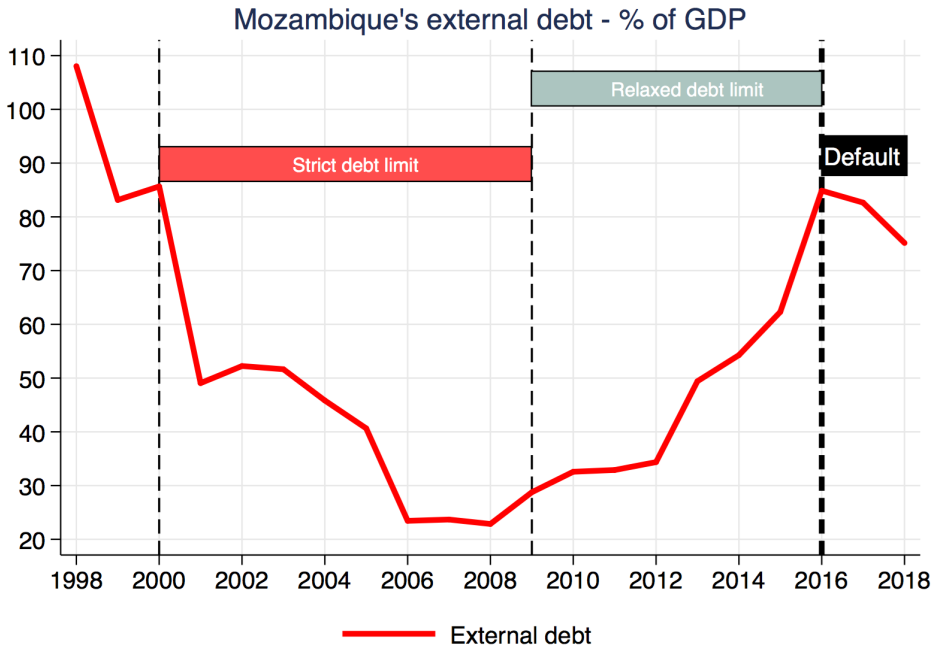


Figure 1: Mozambique's Public Debt

To answer these questions, as a first step, I study the effect of policy relaxation on the external debt-to-GDP ratio. Using a difference-in-differences estimation, with 15 LICs over the period of 2000 to 2018, I show that countries that benefited from policy relaxation seem more likely to experience a significant increase in their external debt-to-GDP ratio, reaching, on average, 90% between 2000 and 2018. Motivated by this result, as a second step, I evaluate the policy quantitatively, using a small open economy model of sovereign default à la Eaton and Gersovitz (1981) - for LICs. Standard debt models mainly focus on developed and emerging economies. However, the debt composition in poor countries is different, since they rely heavily on concessional debt and have little to no access to private international markets (Koeda, 2008). In my model, I include three agents to reflect LICs economies: the government (LIC debtor), and two types of creditors - multilateral institutions that lend at subsidized rates and private lenders that lend at a non-subsidized rate. I find that, when the government is more impatient than the lenders, it will run up debt. It does a good job predicting the country's debt accumulation path and its entry into crisis. Finally, I show that having an impatient government from the perspective of a more patient household leads to a 0.9% decrease in welfare. The model builds up on the work of Cole

and Kehoe (2000) and Conesa and Kehoe (2017) and allows for self-fulfilling debt crises, where the main risk of crisis is coming from the loss of confidence of private lenders. The government is impatient and borrows more than the social optimum. The relaxation of the debt limits policy implemented jointly by the Bank and the IMF created an additional borrowing space. With little to no incentive to reduce debt, the government continues to accumulate it. The lower the government's discount factor is (compared to international lenders), the more likely the country is to enter a crisis zone and find itself in debt distress. The benchmark model consists of an impatient government, multilateral institutions that are risk-neutral, and international lenders, who are also risk-neutral. To keep things simple, there is no representative household's consumption-investment decision as in Conesa and Kehoe (2017). The impatient government smooths consumption by issuing two types of bonds (subsidized and non-subsidized). To reflect the data, I assume that the bonds differ in terms of prices. The price of subsidized debt is higher than the price of non-subsidized debt. Based on this assumption, the government always chooses to sell its subsidized bonds first. Subsidized loan contracts are enforceable, that is, defaulting on these loans will be too costly for the government. Once the borrowing limit is reached for the cheaper loans, the sovereign will tap on the private market. During each period, the government chooses to repay or default on non-subsidized loans. The quantitative contribution of the paper is that bond spreads increase with debt. The farther the government is from the *safe* zone, the higher is the default probability.

In the quantitative analysis, the calibrated model is based on Mozambique's recent default. I discipline the government's discount factor by targeting the average non-subsidized debt ratio, while for the default probability I target the average yield from 2010 to 2018. I use calibrated parameters to solve the model numerically. I, then, compare the benchmark model with an impatient government to that with a less impatient one.

Related Literature. A large body of literature has emerged, Aguiar and Amador (2014) provide a detailed review of the literature on sovereign debt. An incomplete list of examples from said literature includes Sachs (1989), Krugman (1988), Aguiar and Gopinath (2006),Chatterjee and Eyigungor (2012), Aguiar and Amador (2013).

Cole and Kehoe (2000) provide a theory of self-fulfilling debt crises. The authors analyze the optimal policy for a government when such crises can happen to occur. They build a dynamic stochastic general equilibrium model with self-fulfilling crises. They show that, when debt levels are in the crisis zone, then the probability of default is greater than zero. Their main finding

is that the government's optimal response during a crisis is to run down its debt. Conesa and Kehoe (2017) extend the Cole and Kehoe (2000) model by introducing income shocks, through a recession, and recovery is unsure. They find that it is optimal for the government to have fiscal surpluses to reduce debt. They also find that it might be optimal for the government to run up its debt in the crisis zone, however, if the recovery does not occur then the government ends up defaulting on its debt. In contrast to Conesa and Kehoe (2017), I have an impatient government and I introduce two types of debt.

Arellano (2008) provides a quantitative framework to explain the determinants of interest rates in incomplete markets. The author imposes a non-linear functional form for the default costs so that high default probabilities can be calibrated. I model default costs by assuming a linear quadratic function, with a probability default that increases with the level of debt.

This paper is related to the literature on sovereign debt as well as the literature on debt relief policies and debt sustainability in LICs. In Easterly (2002), the author studies the paradox of debt related to heavily indebted poor countries (HIPCs). HIPCs became heavily indebted after two decades of debt relief efforts. He shows that impatience could lead to overborrowing. Koeda (2008) investigates the optimal debt policy for LICs through a concessional lending problem. The paper shows that the LIC tends to accumulate a large amount of concessional debt to smooth consumption rather than for investment purposes. Having access to subsidized loans, the country becomes forever aid dependent. Reinhart and Trebesch (2016) study the dynamics of debt relief and its aftermath. They find that the economic landscape of debtor countries improves significantly after debt relief operations, but only if these involve debt write-offs. Softer forms of debt relief, such as maturity extensions and interest rate reductions, are not generally followed by higher economic growth or improved credit ratings (Reinhart and Trebesch, 2016). Songwe and Awiti (2021) report that at least 48% of African countries had a debt ratio above 70% in 2019. They argue that the speed of debt accumulation matters and that monitoring the speed of debt could alter the path of debt to more sustainable levels. Ndulu and O'Connell (2021) mention that public debt levels in sub-Saharan Africa rose fast in the wake of the 2008 financial crisis, and many countries are now classified by the World Bank and the IMF as at high risk of debt distress.

Outline. The paper is organized as follows. Section 2 describes the relaxation policy and documents that countries that benefited from the intervention see a significant increase in their

external debt-to-GDP ratio. Section 3 discusses the model and its mechanism and Section 4 presents the quantitative results. Section 5 concludes.

2 Motivation

This section presents empirical evidence regarding the interaction between the relaxation of the debt limit policy and external debt.Subsection 2.1 describes the new debt limits policy implemented jointly by the IMF and the World Bank. Subsection 2.2 outlines the data sources used and demonstrates that LICs that benefited from the relaxation policy seem more likely to experience a significant increase in their external debt.

2.1 External Debt Limits Policy

While debt relief leads to a decrease in debt to output ratio, which in turn results in an increase in resources for the recipient country; it also adds a risk of "free-riding".[2] To minimize the free-rider risk, the IMF/World Bank programs with LICs typically include external debt limits. Following the HIPC and MDRI programs, a non-concessional borrowing policy (NCBP) was implemented jointly by the Bank and the Fund in 2006. The NCBP required the external debt contracted or guaranteed by the official sector to include minimum concessionality requirements (typically of 35%). Generally, this would mean a restriction of non concessional external borrowing (NCB) also known as the zero NCB ceiling but no constraint on concessional borrowing. In some cases, exception on the zero NCB ceiling would be made and would allow for looser limits. [3] The policy's main criticism was that the concessionality requirements were constraining the LICs. As a result, the IMF implemented new guidelines in late 2009, followed by the introduction of more flexibility in the NCBP. The new policy goal was to take into account the broad range of situations that the LICs were facing. Meaning that if a country was at a high risk of being in debt distress it should have tighter concessionality requirements than a country with a low risk of being in debt distress. LICs public financial management (PFM) capacity was also a factor, the higher a country's PFM capacity, the more likely it will be able to implement and benefit from looser concessionality

[2]The World Bank's International Development Association (IDA) defines "free-riding" as situations in which debt relief or grants could potentially cross-subsidize lenders that offer non-concessional loans to recipient countries.

[3]See FUND (2006).This was the case until the policy change in 2009.

requirements. Under this new framework, concessionality requirements are divided into groups[4] such that higher capacity countries will have more options while lower capacity countries will continue with the standard or higher concessionality requirements depending on the country's debt vulnerability.

2.2 Empirical Motivation

I use annual data over the period between 2000 and 2018 for 15 low-income countries that benefited from the HIPC program in the early 2000s. The data (285 observations) are obtained from several databases: the World Bank's International Debt Statistics (external debt, external multilateral debt, external bilateral debt, external debt from private creditors),the World Bank's World Development Indicators (current GDP). I use Bloomberg to get yearly prices for bonds issued between 2010 and 2018 for Mozambique. To identify countries that benefited from the debt limit relaxation policy, I use the FUND (2012) policy paper[5]. To get the list of African countries that issued bonds between 2006 and 2014 I use Mecagni et al. (2014) book, which examines the rise in international sovereign bonds issued by SSA economies and countries' central banks.

Table 1: Debt composition of countries with relaxed debt limit and countries with strict debt limit before 2009

Variables	(1) Strict debt limit	(2) Relaxed debt limit	(3) *p-value*
% GDP	Mean	Mean	
External debt (07)	40.56	15.28	*0.15*
Non-subsidized debt (07)	18.06	5.65	*0.12*
Subsidized debt (07)	22.50	9.63	*0.20*
External debt (08)	33.24	14.78	*0.16*
Non-subsidized debt (08)	13.91	4.80	*0.13*
Subsidized debt (08)	19.33	9.98	*0.23*
Number of countries	15	15	

Notes: This table presents descriptive statistics for the sample of analysis. Data come from the World Bank's International Debt Statistics and the World Bank's World Development Indicators

[4]See FUND (2009)"Changing Patterns in Low-Income Country Financing and Implications for Fund Policies on External Financing and Debt".

[5]See "2011 Review of Conditionality" - Table 3.PRGT-Eligible Countries:External Debt Concessionality Requirements Under The New Policy

Table 1 displays the debt composition of countries that benefited from the IMF debt limit policy relaxation in 2009 as well as that of countries with strict debt limit. The table shows external debt, non-subsidized debt and subsidized debt in GDP percent - between the two groups - for the years 2007 and 2008 (prior to the policy implementation). Countries under a strict debt limit seem to hold,on average, more debt as compared to countries that benefited from the policy. However, the difference is not statistically significant (column (3)). Figure 2 shows the debt dynamics for both groups between 2000 and 2018. Notably, the differences are striking. During the debt relief (HIPC and MDRI) window -from 2000 to 2006-, the debt ratio decreased at a fast pace in both groups. From 2008 to 2011, there seemed to be a shift, with the treatment group accumulating debt at a faster pace than the control group. However, the former still held less debt on average than the control group. From 2011 onward, the average debt has been higher in treated countries as compared to the control group.

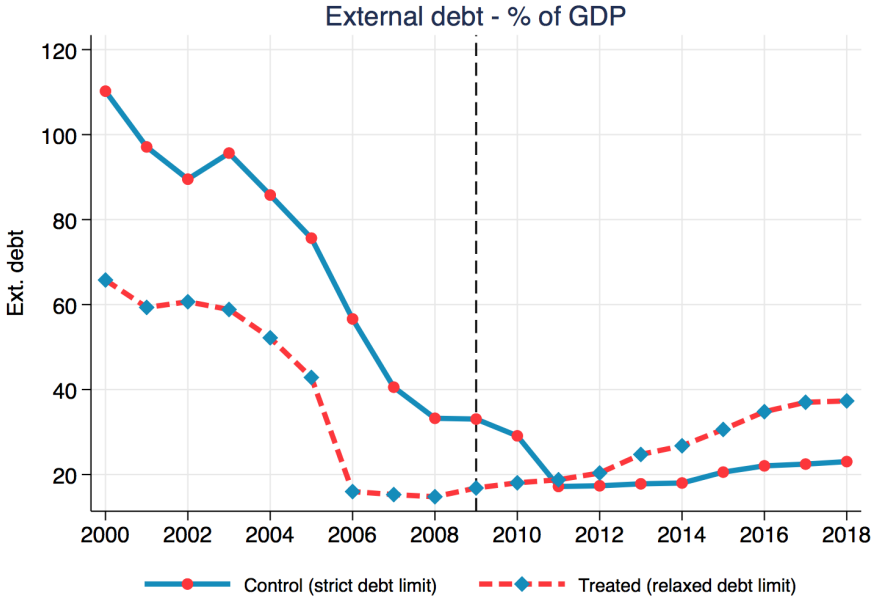


Figure 2: External debt dynamics

To evaluate the IMF policy, I will first estimate a difference-in-differences (DID) regression, as in Reinhart and Trebesch (2016). I look at the impact of the relaxation policy on external debt and I use the relaxation policy as the treatment. Using a two-way fixed effect model, the following equation is estimated using the linear regression:

$$log\ (Debt/GDP)_{it} = \beta_0 + \beta_1 D_{it} + \alpha_i + \eta_t + + \epsilon_{it} \tag{1}$$

where $Debt/GDP_{it}$ is log of debt-to-GDP for country i , D_{it} is an interaction term that equals one for countries benefiting from the policy after 2009 (treated group) and zero for the control group. α_i and η_t are respectively country and time fixed effects. Time t is from 2000 to 2018, while ϵ_{it} is the random error term. The coefficient of interest is β_1, as it captures the impact of the relaxation policy. For example, if the relaxation of the debt limit for a country leads to an increase in debt ratio, then $\beta_1 > 0$. The country fixed effects, α_i, take into account country time-invariant heterogeneity. The time fixed effects, η_t, capture a common time trend.

As it is the case with difference-in-differences models, parallel trends are assumed, meaning that the only difference before and after the policy between the two groups is coming from the relaxation of debt limit. Therefore, if countries did not experience a relaxation of their debt limit, the trend on debt-to-GDP should have been similar in both groups. In this light, I use an event study to look at common trends before and after treatment. I run the following regression:

$$log\ (Debt/GDP)_{it} = \rho_0 + \sum_{k=2007}^{2003} \rho_1 treat_{ik} + \sum_{k=2009}^{2013} \rho_1 treat_{ik} + \alpha_i + \eta_t + + \epsilon_{it} \tag{2}$$

where $Debt/GDP_{it}$ is log of debt-to-GDP for country i, $treat_{ik}$ are dummy variables for the time relative to treatment. As in equation (1) α_i and η_t are respectively country and time fixed effects, while ϵ_{it} is the random error term.

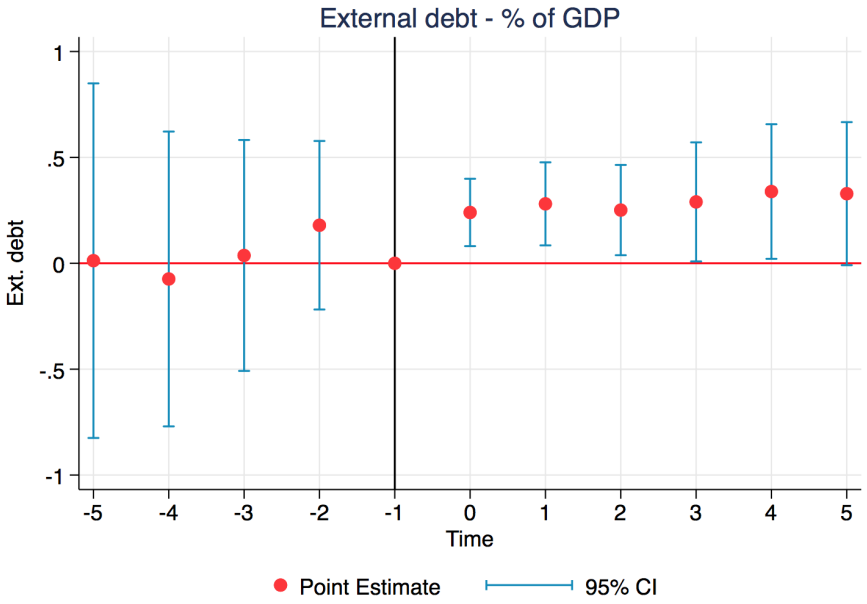


Figure 3: Event study

The results of the event study, depicted in Figure 3, show the point estimates and confidence intervals of the regression (equation 2) for each time period before and after the treatment period. On the horizontal axis we plot the time dummies. For instance, if treatment happened in 2010 and the observation's period (current period in the data) is also 2010, then $t = 0$. If instead of 2009, the treatment occurred in 2011, then $t = 1$. The vertical line is a 95% confidence interval, the red horizontal line is set at 0 and the point of reference is at $t = -1$. Standard errors are clustered at the country level. Except for $t = -1$, the pre-treatment coefficients (ρ_1) are close to zero and are all statistically insignificant. An anticipation of treatment might explain the non-zero coefficient at $t = -1$.

Table 2 reports the preliminary results of equation (1). Columns (1), (2) and (3) show the results of the linear regression with log of external debt-to-GDP, log of non-subsidized debt-to-GDP and log of subsidized debt-to-GDP as outcome variables, respectively. The analysis shows that, countries that benefited from the policy relaxation are more likely to experience an increase in their log external debt-to-GDP ratio(column (1)) by 0.65 or 93%, on average, between 2000 and 2018. Looking at columns (2) and (3) we can see the increase is coming from non-subsidized loans with a significant increase of 0.94 (or 150%). Subsidized loans also increase but the results are not statistically significant. This is a drastic change compared to the late 80s and early 90s when the bulk of debt was mainly subsidized. It is important to notice that these are preliminary results

and there might be omitted variables that may be creating an estimation bias for the coefficients.

Table 2: Impact of relaxation policy on log of debt-to-GDP ratio

Outcome variables	External Debt	Non-subsidized debt	Subsidized Debt
	(1)	(2)	(3)
Post 09 x Treated group	**0.654****	**0.941*****	0.414
	(0.254)	(0.305)	(0.249)
Intercept	4.305***	3.159***	3.762***
	(0.0962)	(0.078)	(0.1279)
Observations	285	285	285
R^2	0.462	0.193	0.534
Number of countries	15	15	15

Standard errors clustered at the country level in parentheses
$***p < 0.01$, $**p < 0.05$, $*p < 0.1$

3 Model

Environment: I consider a small open economy with constant exogenous output $\bar{y}$ and time is discrete. There are three agents: the government, the multilateral institutions,and the international lenders. To keep things simple, there is no representative household's consumption-investment decision as in Conesa and Kehoe (2017). I assume that the government tax rate is equal to zero, with the consumption of the representative household in this economy being equal to the government spending, however the consumption decision is made by the government. Following Aguiar et al. (2020) the government's discount factor (β_g), is different than the household discount factor (β_h). The government is said to be impatient when $\beta_g < \beta_h$. The government has access to two types of loans: subsidized, B^s, at price q^s through the multilateral institutions, and non-subsidized, B^{ns}, at price q^{ns} through the international lenders. The only difference between those two loans is the yield. Each period the government chooses how much debt it can borrow the next period. Subsidized debt contracts are enforceable and the government cannot default on that debt, however, it can choose to repay or default on its non-subsidized debt. Both, the multilateral institutions and the international lenders are risk-neutral agents.

The aggregate state of the economy is $\mathbf{s} = (B^s, B^{ns}, \zeta)$ with the current period level of subsidized and non-subsidized debt, ζ is the sunspot variable. The timing at each period is similar to Conesa and Kehoe (2017):

1. The aggregate state $\mathbf{s} = (B^{ns}, B^s, \zeta)$ is observed. Given bonds prices, the government chooses next period subsidized and non-subsidized debts (B'^s and B'^{ns}).

2. Given bonds prices, the multilateral institutions and international lenders choose how much debt to buy, respectively b'^s and b'^{ns}.

3. The government chooses to repay or default on its non-subsidized debt B^{ns}.

4. Then, the government consumes g.

I assume that the price of the subsidized bonds, q^s, is greater than the price of the non-subsidized loans q^{ns}, as observed in the data. Multilateral lenders often lend at concessional rates lower than on the private markets.

Proposition 1: *This implies that the government will always choose to borrow on concessional*

terms (B'^s) until they hit the borrowing limit, $B'^s = \bar{b}^s$. Only then that the government will choose to tap on the non-subsidized debt B'^{ns}.

Proof. See appendix.

Multilateral institutions: are risk-neutral agents with discount factor β_s. They face the following maximization problem:

$$W_s(b^s, B'^s, \mathbf{s}) = \max_{(X_s, b'^s)} \left\{ X_s + \beta_s W_s(b'^s, B''^s, \mathbf{s'}) \right\} \tag{3}$$

s.t.

$$X_s + q^s(B'^s, \mathbf{s}) b'^s = w_s + b^s$$

$$b^s \leq \bar{A}$$

where w_s is the income, X_s the consumption level with the constraint $b^s \leq \bar{A}$ not allowing for Ponzi schemes.From the maximization problem, we derive the bonds prices: $q^s = \beta_s$

International lenders: are risk neutral with discount factor β_{ns}. They face the following maximization problem:

$$W_{ns}(b^{ns}, B'^{ns}, \mathbf{s}) = \max_{(X_{ns}, b'^{ns})} \left\{ X_{ns} + \mathbb{E}\left[\beta_{ns} W_{ns}(b'^{ns}, B''^{ns}, \mathbf{s'})\right] \right\} \tag{4}$$

s.t.

$$X_{ns} + q^{ns}(B'^{ns}, \mathbf{s}) b'^{ns} = w_{ns} + z(q^{ns}(B'^{ns}, \mathbf{s}), \mathbf{s}) b^{ns}$$

$$b^{ns} \leq \bar{A}$$

where w_{ns} is the income, z is a dummy variable that equals 0 if default happens, and X_{ns} the consumption level with the constraint $b^{ns} \leq \bar{A}$ not allowing for Ponzi schemes.
From the maximization problem, we get the bond prices, it is equal to the lenders discount factor, adjusted by the probability of default δ:

$$q^{ns}(B'^{ns}, \mathbf{s}) = \beta_{ns}[1 - \delta(B'^{ns})] \tag{5}$$

The default's probability is endogenous to the model, but instead of varying with income shocks, it depends on how much debt the government accumulates while in the crisis zone.

Government: The government faces the following problem.

$$V(\mathbf{s}) = \max_{(g,B'^s,B'^{ns})} \left\{ log(g) + \beta_g \mathbb{E}\left[V(\mathbf{s'})\right] \right\} \tag{6}$$

s.t.

$$g = y + q^{ns}(B'^{ns}, \mathbf{s})B'^{ns} - zB^{ns} + q^s(B'^s, \mathbf{s})B'^s - B^s$$

$$B^s \leq \bar{b}^s$$

$$B^{ns} \leq \bar{b}^{ns}$$

where $0 < \beta_g < 1$ is the discount factor and g is the government's consumption. B^s and B^{ns} are the current period level of subsidized and non-subsidized debt, while B'^s and B'^{ns} are the next period level of debt. q^s and q^{ns} are the bonds' prices.The country's output, y, is the exogenous and of the form: $y = Z^{1-z}\bar{y}$, where z is a dummy variable that equals 0 if default and Z the drop in productivity if default happens.

With the government being impatient, it will end up borrowing up to the debt limit for both types of loans. We will focus on the non subsidized debt since there is no commitment. Non subsidized debt falls into three possible zones:

- Safe zone: when debt is small, the government will never default ($\delta(B'^{ns}) = 0$),
- Crisis zone: when debt is at an intermediate level, the government might default ($0 < \delta(B'^{ns}) < 1$)
- Default zone: when debt is high, the government will default ($\delta(B'^{ns}) = 1$).

In Cole and Kehoe (2000) self-fulfilling crisis arises when there are two possible equilibrium outcomes, one in which the government is able to sell new debt at a positive price and chooses to repay the old debt and another in which the government is unable to sell new debt at a positive price and defaults on the existing debt. Similarly in this environment, if $\zeta > \delta(B'^{ns})$ or/and B'^{ns}

is small enough, the international lenders do not expect the government to default. Conversely if $\zeta \leq \delta(B'^{ns})$ and debt is at an intermediate level (crisis zone), the lenders expect the government to not commit to repay their debt and therefore they are not willing to lend. However if B'^{ns} is too high, the government defaults and the bankers do not lend. This implies that bonds prices:

$$q^{ns}(B'^{ns}, \mathbf{s}) = \begin{cases} \beta_{ns} & if\ B'^{ns} \leq \bar{B}^{ns}\ (safe\ zone) \\ \beta_{ns}(1-\delta(B'^{ns})) & if\ \bar{B}^{ns} < B'^{ns} \leq \bar{b}^{ns}\ (crisis\ zone). \\ 0 & if\ \bar{b}^{ns} < B'^{ns}\ (default) \end{cases} \quad (7)$$

with $\bar{B}^{ns}$ and $\bar{b}^{ns}$ being, respectively, the two cutoff levels of non subsidized debt.

Recursive equilibrium: In this economy, the government lacks commitment on non-subsidized debt and all three agents act sequentially. Given the aggregate state $\mathbf{s} = (B^s, B^{ns}, \zeta)$, the policy functions for the government $B'^s(\mathbf{s})$, $B'^{ns}(\mathbf{s})$, $z(q^{ns}, q^s, B'^{ns}, \bar{b}^s, \mathbf{s})$ and $g(q^{ns}, q^s, B'^{ns}, B'^s, \mathbf{s})$, the prices of subsidized and non-subsidized bonds $q^s(B'^s, \mathbf{s})$, $q^{ns}(B'^{ns}, \mathbf{s})$, give us the equilibrium.

A recursive equilibrium for this economy is defined by a value function $V(\mathbf{s})$, policy functions $B'^s(\mathbf{s})$, $B'^{ns}(\mathbf{s})$, $z(\mathbf{s})$ and $g(q^{ns}, q^s, B'^{ns}, B'^s, \mathbf{s})$ for the government; a value function $W_s(b^s, B'^s, \mathbf{s})$ and a policy correspondence $b'^s(\mathbf{s})$ for the multilateral institutions; a value function $W_{ns}(b^{ns}, B'^{ns}, \mathbf{s})$ and a policy correspondence $b'^{ns}(\mathbf{s})$ for the lenders and finally a system of prices $q^s(B'^s, \mathbf{s})$, $q^{ns}(B'^{ns}, \mathbf{s})$ such that:

1. Given the policy functions $z(q^{ns}, q^s, B'^{ns}, \bar{b}^s, \mathbf{s})$, $g(\mathbf{s})$ and the prices of the bonds $q^s(B'^s, \mathbf{s})$, $q^{ns}(B'^{ns}, \mathbf{s})$; $V(\mathbf{s})$ and policy functions $B'^s(\mathbf{s})$, $B'^{ns}(\mathbf{s})$ solve the government's problem at the beginning of the period:

$$V(\mathbf{s}) = \max_{(g, B'^s, B'^{ns})} \left\{ log(g) + \beta_g \mathbb{E}\left[V(\mathbf{s'})\right] \right\} \quad (8)$$

s.t.

$$g(q^{ns}, q^s, B'^{ns}, B'^s, \mathbf{s}) = y(z) + q^{ns}(B'^{ns}, \mathbf{s})B'^{ns} - z(q^{ns}, q^s, B'^{ns}, \bar{b}^s, \mathbf{s})B^{ns} + q^s(B'^s, \mathbf{s})B'^s - B^s$$

2. $b'^{ns}(b^{ns}, B'^{ns}, \bar{b}^s, \mathbf{s})$ solve the international lenders' problem and $q^{ns}(B'^{ns}, \mathbf{s})$ satisfies the

lenders' no arbitrage condition:

$$q^{ns}(B'^{ns}, \mathbf{s}) = \beta_{ns}[1 - \delta(B'^{ns})] \tag{9}$$

3. $b'^s(b^s, B'^s, \mathbf{s})$ solve the multilateral institutions' problem and $q^s(B'^s, \mathbf{s})$ satisfies the multilateral' no arbitrage condition:

$$q^s(B'^s, \mathbf{s}) = \beta_s$$

4. Given $V(\mathbf{s})$; the policy functions $z((q^{ns}, q^s, B'^{ns}, \bar{b}^s, \mathbf{s})$, $g(q^{ns}, q^s, B'^{ns}, B'^s, \mathbf{s})$ solve the government's problem at the end of the period:

$$\max_{(g, B'^s, B'^{ns})} \left\{ log(g) + \beta_g \mathbb{E}\left[V(\mathbf{s'})\right] \right\} \tag{10}$$

s.t.

$$g = y(z) + q^{ns} B'^{ns} - zB^{ns} + q^s B'^s - B^s$$

With the government running up its subsidized debt, B^s, in T periods, we solve for the optimal policy functions.

To solve for the lower bound of debt for the crisis region, $\bar{B}^{ns}$, we need to define two values functions. Let V_{run} be the value of running up debt when price of of next period non subsidized debt, B'^{ns}, is 0:

$$V_{run}(B^s, 0) = log[\bar{y} - \bar{B}^{ns} + \bar{b}^s(\beta_s - 1)] + \frac{\beta_g}{1 - \beta_g} V(\bar{b}^s, 0) \tag{11}$$

Let V_d be the discounted value of default:

$$V_d = \frac{1}{1 - \beta_g} log[Z\bar{y} + \bar{b}^s(\beta_s - 1)] \tag{12}$$

Equating $V_{run}(B^s, 0)$ and V_d, allows us to obtain $\bar{B}^{ns}$ analytically:

$$\bar{B}^{ns}(Z, \beta_g, B^s) = [\bar{y} + \bar{b}^s(\beta_s - 1)] - \frac{[Z\bar{y} + \bar{b}^s(\beta_s - 1)]^{\frac{1}{1-\beta_g}}}{[\bar{y} + \bar{b}^s(\beta_s - 1)]^{\frac{\beta_g}{1-\beta_g}}} \tag{13}$$

The upper bound of debt for the crisis region, $\bar{b}^{ns}$,is solved numerically by equating the value of repaying the debt when in the crisis zone to the discounted value of default.
Let $V_r(s)$ be the value of repaying when the price of B'^{ns} is $\beta_{ns}(1-\delta)$:

$$V_r(s) = log[\bar{y} + \beta_{ns}(1-\delta)B'^{ns} - B^{ns} + \bar{b}^s(\beta_s - 1)] + \beta_g \mathbb{E}\left[V_r(s')\right] \tag{14}$$

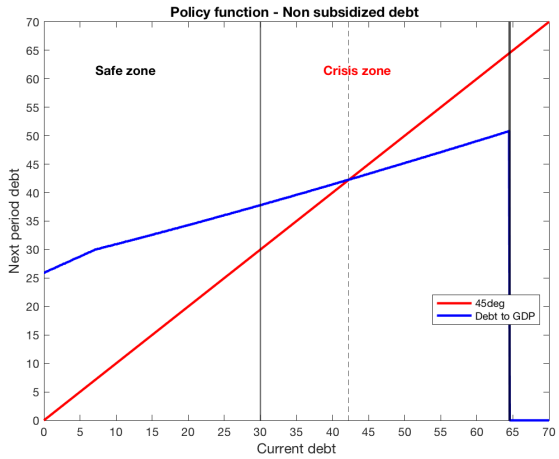
Let V_d be the same discounted value of default:

$$V_d = \frac{1}{1-\beta_g} log[Z\bar{y} + \bar{b}^s(\beta_s - 1)]$$

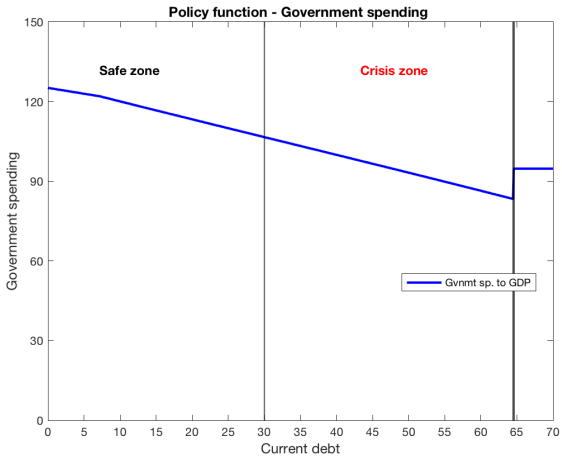
3.1 Policy Functions

This section describes the mechanism of the model through the main policy functions of the government. Figure 4 plots the policy functions with the debt cutoffs. the lower bound of the crisis zone ($\bar{B}^{ns}$) is about 30% of GDP. Below that threshold the government is in the safe zone and it is not optimal to default. Above that cutoff, the government is in the crisis zone and a self-fulling crisis might happen. The upper bound of the crisis zone ($\bar{b}^{ns}$) is around 65% of GDP. Above that level it is optimal for the government to default. The optimal policy for the government is to accumulate debt fast in the safe region, since bond prices (Figure 4d) are constant and at their highest. The government keeps accumulating debt in the crisis zone but at a lower speed since default probabilities (Figure 4c) kick in and investors are becoming nervous. If debt level is above 42% of GDP, the government optimal policy is to decrease its debt.

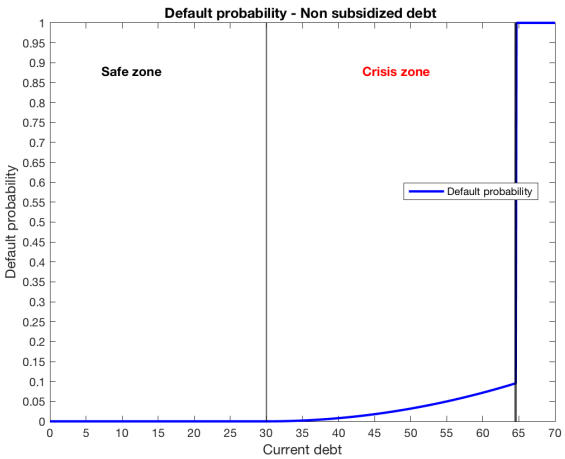
Suppose now that we have a change in bond prices. Figure 5 show how the government adjusts its borrowing policy when bond prices are low (Figures 5a and 5b) versus when prices are high (Figures 5c and 5d). The first thing to notice is that the upper bound of the crisis region ($\bar{b}^{ns}$) decreases when prices are low (39% vs 65% of GDP in the baseline model)and increases when prices are high (90%). When prices are extremely low, the optimal policy for the government is still to accumulate debt fast in the safe region, once debt level hits the lower bound of the crisis zone, it stays there. When prices are neither extremely high or low, the government enters the crisis region. Depending on the prices, when debt level is higher than 40% -low prices- or 70% -high prices- (vs 42% of GDP in the baseline model) the optimal policy for the government is to decrease its debt. On the other hand, when prices are extremely low the optimal policy for the government is to accumulate debt fast until it reaches the upper bound of the crisis zone (90%) and stay there.



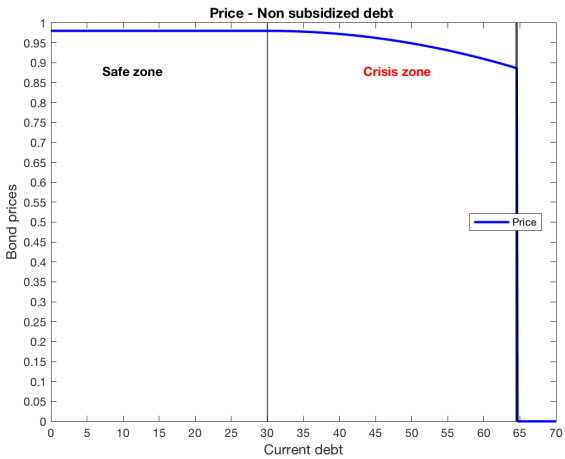
(a) Non subsidized debt



(b) Government's consumption

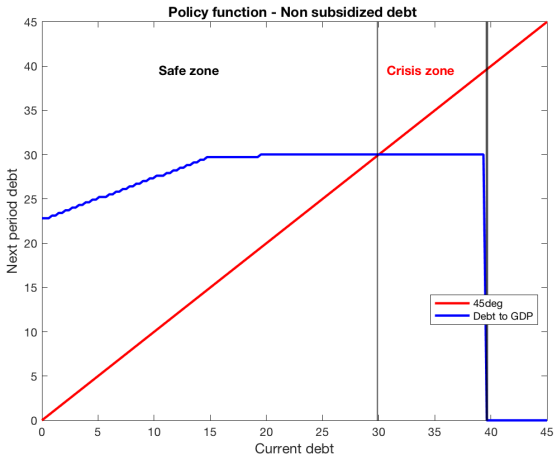


(c) Default probabilities

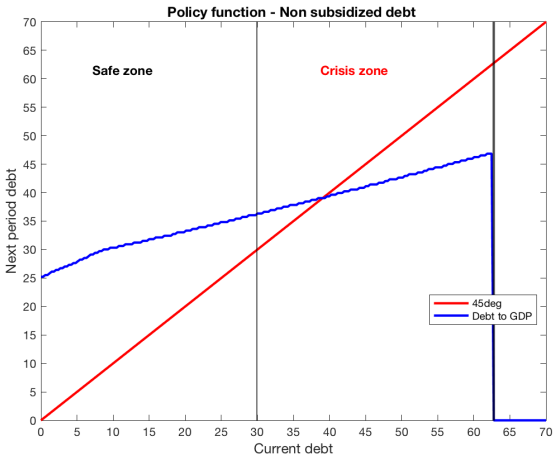


(d) Bond prices

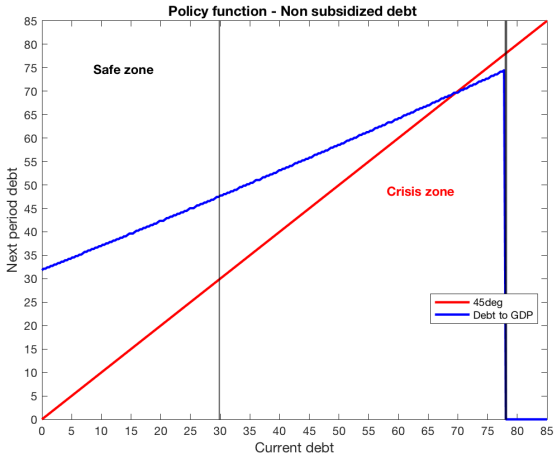
Figure 4: Policy functions of key variables



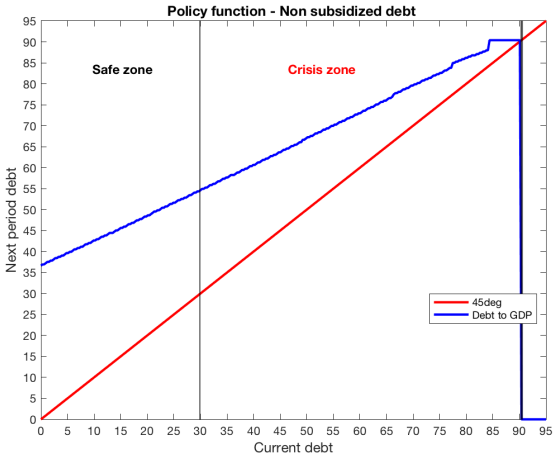
(a) Extremely high default probability



(b) High default probability



(c) Low default probability



(d) Extremely low default probability

Figure 5: Non subsidized debt

4 Quantitative Analysis

This section illustrates the quantitative results of the model.

4.1 Calibration

Functional forms: The utility function is of the form $U(g) = log(g)$. Following Hördahl and Tristani (2013), I use a linear-quadratic function, to measure the relationship between default risk and debt. It allows for more volatility in the crisis zone even without exogenous shocks. The probability of default is increasing with the distance of the level of non subsidized debt relative to the lower bound of the crisis zone. I assume that the probability of default has the following functional form:

$\delta(B'^{ns}) = \bar{\epsilon}(B'^{ns} - \bar{B}^{ns})^2$ with $\bar{\epsilon}$ the constant slope.

$$\delta(B'^{ns}) = \begin{cases} 0 & if\ B'^{ns} \leq \bar{B}^{ns}\ (safe\ zone) \\ \delta(B'ns) & if\ \bar{B}^{ns} < B'^{ns} \leq \bar{b}^{ns}\ (crisis\ zone). \\ 1 & if\ \bar{b}^{ns} < B'^{ns}\ (default) \end{cases}$$

Parameters values: The model is solved numerically to evaluate its quantitative predictions regarding the non subsidized debt and the government's default decision. The benchmark model is calibrated using Mozambique macroeconomic data from 2006 to 2018. A period in the model corresponds to a year in the data. All the parameter values (Table 4) are observed in the data. The government's discount factor β_g and the slope of the average default risk ($\bar{\epsilon}$) however are chosen such that the model estimations match relevant moments in the data(Table 3).

The average subsidized interest rate match the existing official concessional lending practice of 0.8% (Koeda, 2008) which leads to a multilateral discount factor, $\beta_s = 0.99$. The risk-free interest rate of 2% is set to match the yield of safe bonds as do Conesa and Kehoe (2017), and thus a lenders' discount factor of 0.98. The default penalty $1 - Z$ of 5% is standard in the literature (Alonso-Ortiz et al., 2017). The initial level and debt limit of the subsidized debt come from the

data for years 2006 and 2010. In the model, the government borrows up to the subsidized debt limit before tapping on the non-subsidized debt. The starting year of the non-subsidized debt is set for 2010 since the debt limit relaxation policy was implemented in late 2009. I assume that the initial level of non-subsidized debt is zero. The government's discount factor is disciplined by the average non-subsidized debt to output ratio. The average default risk ($\bar{\epsilon}$) is chosen such that it matches the average yield of non-subsidized bonds, from 2010 to 2016 in the data.

Table 3: Mozambique - Parameters calibrated from the simulations

Description	Parameter	Value	Source/Target
Government's discount factor	β_g	*0.86*	Mean NS debt-to-GDP (27.7% - From 2010 to 2018)
Constant	$\bar{\epsilon}$	$8 * 10^{-5}$	Mean NS yield (11% - From 2010 to 2016)

Table 4: Mozambique - Parameters calibrated from the data

Description	Parameter	Value	Source/Target
Multilateral's discount factor	β_s	0.99	Koeda (2008)
Lenders' discount factor	β_{ns}	0.98	Conesa and Kehoe (2017)
Output	$\bar{y}$	100	Arbitrary value
Drop-in productivity	Z	0.95	Conesa and Kehoe (2017)
Initial level of subsidized debt	B^s	12	NS debt-to-GDP in 2006
Subsidized debt limit	$\bar{b^s}$	23	NS debt-to-GDP in 2010
Initial level of non subsidized debt	B^{ns}	0	Arbitrary value

4.2 Quantitative Results

Mozambique: The calibrated model predicts that an impatient government will enter the crisis zone a year earlier than a less impatient government on average.

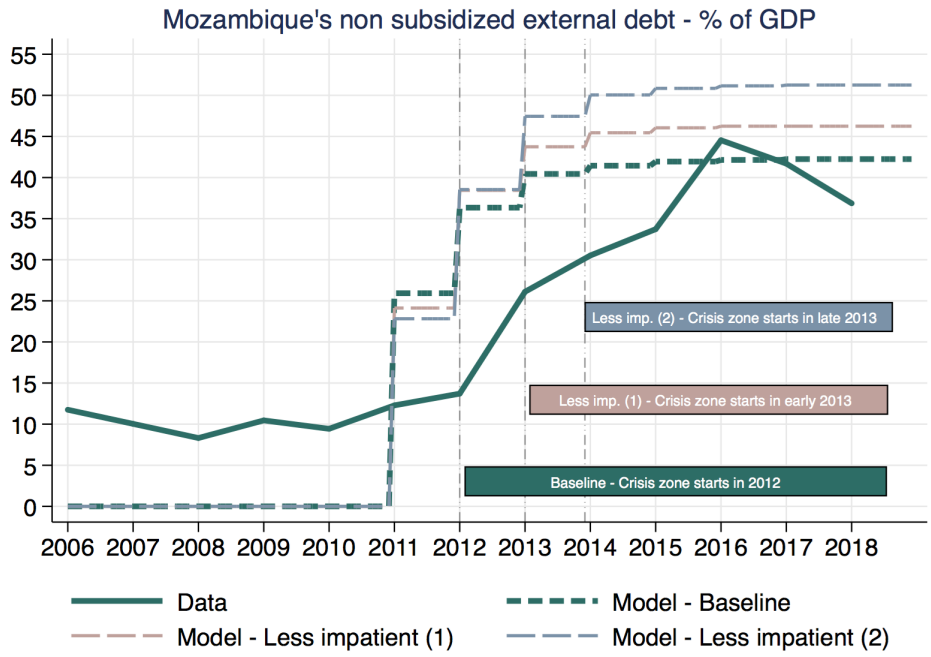


Figure 6: Non subsidized debt

4.3 Welfare Analysis

Now I explore the question of whether having a government in disagreement with the households, leads to a potential welfare gain or loss. Let β_h be the household discount factor such that: $\beta_g < \beta_h = \beta_{ns}$. To quantify the welfare gains of debt accumulation, I compare the benchmark model (impatient government) with a less impatient government. Table 5 shows that households are better off when the discount factor of the government is closer to the lenders'.

Table 5: Impatient Government vs Patient Government

	Baseline: $\beta_g = 0.86$	$\beta_g = 0.91$
Mean of NS debt B^{ns}	38.73	43.41
$\bar{B}^{ns}$	30.00	40.10
$\bar{b}^{ns}$	64.56	80.58
Change in welfare (λ)	-0.9%	-

To compute the welfare losses of having an impatient government, I proceed as follows. Given the governments' policy functions, I compute the consumption paths for both economies and take $N = 1000$ draws $G = \{\{(g_t{}^{imp}, g_t{}^{pat})\}_{t=1}^{T=1000}\}_{n=1}^{N=1000}$. Let W^{imp} be the value of having an impatient government such that:

$$W^{imp} = \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta_h^t \left[log(g_t^{imp}) \right] \right\}$$

and define W^{pat} as the value of having a patient government such that:

$$W^{pat} = \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta_h^t \left[log((1+\lambda) g_t^{pat}) \right] \right\}$$

Let λ be the welfare losses that the households encounter from having an impatient government. By equating the two equations, W^{imp} and W^{pat}, we solve for λ:

$$\lambda = exp\left\{ \frac{W^{imp} - \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta_h^t \left[log(g_t^{pat}) \right] \right\}}{\sum_{t=0}^{\infty} \beta_h^t} \right\} - 1$$

I estimate a loss in welfare of 0.9% if the government is less impatient ($\beta_g = 0.91$).

5 Conclusion

This paper documents that many recipient countries of debt relief programs experience a fast resurgence in their levels of external public debt. First, I study the effect of policy relaxation on non-subsidized borrowing implemented in 2009 by the World Bank and the IMF. Using a difference-in-differences estimation, with 15 LICs, I show that countries that benefited from policy relaxation, seem more likely to experience a significant increase in their non-subsidized debt. Secondly, I use a quantitative sovereign default model enhanced with three agents to reflect LICs economies: the government (LIC debtor) and two types of creditors, multilateral institutions that lend at subsidized rates and private lenders that lend at non-subsidized rates. In this environment, the risk of debt crisis stems from the loss of confidence among private lenders. This set-up may capture important characteristics of Mozambique's 2016 financial crisis. The quantitative contribution of the paper is that the spreads increase with rising debt. The model predicts that a government that is more impatient than non-subsidized lenders runs up its debt. The more impatient is the government, the higher is the probability of a default and the larger is the welfare loss for the citizens of the country.

For poor countries, defaulting often means the loss of access to financial markets or/and suspension of aid/relief programs, as a result of which the country cannot borrow anymore. Repaying the loans means imposed austerity on a population already living in extreme austerity. With the COVID-19 crisis, the debt burden of SSA countries is likely to increase. There are several important implications for policy makers. First, with LICs gaining access to the private market, more coordination between the government, the private lenders and the multilateral institutions should be implemented. Secondly, multilateral institutions should enforce more transparency, especially after large-scale debt relief programs such as the HIPC and MDRI. Regarding future guidelines on debt limit relaxation for LICs, perhaps a more gradual approach should be implemented in the future, especially for less diversified economies. Finally it is key to incentivize governments in those countries to implement sound policies.

Overall, this paper provides a tractable framework to study the debt accumulation in low-income nations with a government in disagreement with the households. An interesting extension of the paper would be to introduce long term maturity debt as in Conesa and Kehoe (2017).

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A The HIPC initiative and MDRI

Launched in 1996, the original Initiative for Heavily Indebted Poor Countries (HIPCs) marked the first time that multilateral, Paris Club, and other official bilateral and commercial creditors united in a joint effort to reduce the external debt of the world's most debt-laden poor countries to "sustainable levels"—that is, levels that allow these countries to service their debt through export earnings, aid, and private capital inflows without compromising long-term, poverty-reducing growth. Assistance under the HIPC initiative is limited to countries that have per capita incomes low enough (GNI per capita less than $2390) to qualify for World Bank and IMF concessional lending facilities, and that face unsustainable debt burdens even after traditional debt relief. The vast majority of beneficiary countries are in Africa.

The main critique of the original HIPC initiative was that the sustainability targets were set in light of the empirical work that had examined largely middle income countries (MICs) while LICs had less capacity to sustain external debt. In 1999, a review of the HIPC initiative was carried out by the World Bank and the IMF in broad consultation with civil society organizations and public officials. As a result, the international community agreed to enhance the HIPC initiative and committed to providing faster, broader, and deeper debt relief. With the qualifying thresholds being lowered, more countries were eligible for debt relief, while some were eligible for more debt relief. The country eligibility is determined as follows:

Decision Point - Stage 1: The HIPC needs to establish a three year (or less) track record of good macroeconomic performance. Once that goal is attained, the country is considered to have reached its "decision point". The country's eligibility and the amount of debt relief are then determined by the IMF and World Bank Boards. Debt relief and other assistance now begin flowing as soon as the decision point is reached, with the amount based on the country's immediate needs and capacity for channeling the funds to poverty-reducing purposes.

Decision Point - Stage 2: The HIPC needs to establish another track record by implementing the policies determined at the "decision point".

Completion Point: The international community commits to provide sufficient assistance by a particular date (the "completion point") in an amount that would enable the country to achieve debt sustainability. At the "completion point," the remainder of the full stock-of-debt reduction pledged is delivered. (IMF website)

As of February 2020, thirty six countries are at post-completion point.

The MDRI: In June 2005, the Group of 8 (G8) major industrial countries proposed that three multilateral institutions—the IMF, the International Development Association (IDA) of the World Bank, and the African Development Fund (AfDF)—cancel 100 percent of their debt claims on countries that had reached, or would eventually reach, the completion point under the enhanced HIPC Initiative. The goal of the MDRI was to provide full debt relief to free up additional resources to help these countries reach the United Nations millenium development goals.(IMF website)

Combined, the MDRI and HIPC initiative have provided around $99 billion in debt relief.